

Gravitational waves in preheating

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We study the evolution of gravitational waves through the preheating era that follows inflation. The oscillating inflaton drives parametric resonant growth of scalar field fluctuations, and although super-Hubble tensor modes are not strongly amplified, they do carry an imprint of preheating. This is clearly seen in the Weyl tensor, which provides a covariant description of gravitational waves.

1. INTRODUCTION

Gravitational waves in inflationary cosmology are produced by sub-Hubble scale vacuum fluctuations. They are stretched to super-Hubble scales by inflationary expansion, and then they generate anisotropies in the cosmic microwave background (CMB) (see, e.g., Grishchuk 1975, Mukhanov et al. 1992). Scales which re-enter before matter-radiation equality will rapidly redshift (and suffer small damping during recombination), having negligible effect on CMB temperature anisotropies, while scales which re-enter later can affect the temperature anisotropies at large angles. Roughly speaking, we can approximate the impact of gravitational waves upon CMB temperature anisotropies by neglecting all local causal dynamics and treating the large-scale fluctuations as frozen beyond the Hubble scale until re-entry, where their amplitude is conserved and determined by the inflationary Hubble rate at the time of Hubble-crossing. This picture is confirmed by the evolution equation (Lifshitz 1946)

$$f''_{ij} + 2\frac{a'}{a}f'_{ij} + k^2 f_{ij} = 0 \quad (1)$$

for the Fourier modes of the transverse traceless tensor perturbation f_{ij} , defined (on a flat background) by

$$ds^2 = a^2(\eta) [-d\eta^2 + (\delta_{ij} + f_{ij}) dx^i dx^j] . \quad (2)$$

Equation (1) shows that on very large scales, $k/aH \ll 1$ (where $H = a'/a^2$),

$$f_{ij} \approx A_{ij} + B_{ij} \int \frac{d\eta}{a^2} , \quad (3)$$

where $A'_{ij} = 0 = B'_{ij}$. If $a' > 0$ then the B -mode is decaying, and we have $f_{ij} \approx \text{constant}$, with the constant determined by the Hubble rate at inflationary Hubble-crossing, $H(\eta_c) = k/a(\eta_c)$.

This simple picture is modified by small corrections induced at preheating. At the end of slow-roll inflation, the inflaton oscillates and transfers its energy to fluctuations, initiating the reheating era, which ends when created particles thermalize as a radiative plasma. Reheating is often initiated by a preheating era, marked by coherent inflaton oscillations which drive parametric resonant amplification of scalar fluctuations (see, e.g., Traschen and Brandenberger 1990, Kofman et al. 1994, 1997). One crucial point about preheating is that, since the inflaton is coherent on scales well beyond the Hubble horizon, it is in principle possible for super-Hubble fluctuations to be amplified *without violating causality* (Bassett et al. 1999a,b).¹ In the case of scalar fluctuations of the *metric*, this can in principle produce nonlinear amplification, depending on initial conditions and coupling strengths (Bassett et al. 1999a,b,c, 2000, Ivanov 2000, Jedamzik and Sigl 2000, Liddle et al. 2000). While tensor fluctuations will not be strongly amplified by scalar inflaton oscillations, these oscillations nevertheless could leave an imprint on the tensor spectrum on large scales. Such a possibility is usually ruled out on causality grounds, but no such causality constraint operates during coherent oscillations of the inflaton condensate.

¹It is also possible for small-scale gravitational waves to be generated by gravitational bremsstrahlung via rescattering of scalar fluctuations during preheating (Khlebnikov and Tkachev 1997). This is a particular example of the generation of tensor perturbations by scalar perturbations at second order (Matarrese et al. 1998).

These small corrections will be carried into Eq. (1) via the scale factor, which inherits an oscillatory addition to its average value. However, the nature of the effect is more clearly brought out via an alternative description of gravitational waves, based on the idea that a full characterization requires the curvature tensor, not the metric (Pirani 1957). Transverse traceless modes are given by the divergence-free electric and magnetic parts of the Weyl tensor

$$E_{\mu\nu} = C_{\mu\alpha\nu\beta} u^\alpha u^\beta, \quad H_{\mu\nu} = {}^*C_{\mu\alpha\nu\beta} u^\alpha u^\beta,$$

where u^μ is the background four-velocity (there are no velocity perturbations for tensor modes). This covariant description of gravitational waves was used by Hawking (1966), and is remarkably analogous to electromagnetic radiation theory (Dunsby et al. 1997, Maartens and Bassett 1998).

In this paper, we use the Maxwell-Weyl approach to gravitational waves to investigate the effects of inflaton oscillations in some simple preheating models, generalizing previous work in Minkowski spacetime (Bassett 1997). In Sections 2 and 3, we give the basic equations and their qualitative properties. In Section 4 we present the numerical calculations, and Section 5 contains concluding remarks and discussion.

2. BACKGROUND DYNAMICAL EQUATIONS

The background inflaton is governed by the Klein-Gordon equation

$$\ddot{\varphi} + 3H\dot{\varphi} + V_\varphi = 0, \quad (4)$$

where $V_\varphi = \partial V/\partial\varphi$. We will consider the simple chaotic inflation potentials $V = \frac{1}{2}m^2\varphi^2$ and $V = \frac{1}{4}\lambda\varphi^4$. Although the resonance in scalar fluctuations is dramatically increased when the inflaton is coupled to other fields, for example via the additional potential term $\frac{1}{2}g^2\varphi^2\chi^2$, this does not seem to have an effect on *tensor* fluctuations (Tilley 2000). Thus we will confine ourselves to simplified single-field models of preheating. Preheating in more realistic models ends when backreaction effects of the fluctuations destroy the coherence of inflaton oscillations. In our simplified models, backreaction effects are not incorporated, but we can use the results from detailed investigations to estimate the time that preheating lasts (see, e.g., Kofman et al. 1997).

The Hubble rate is determined by the Friedmann equation

$$H^2 = \frac{1}{3}\kappa^2 \left[\frac{1}{2}\dot{\varphi}^2 + V(\varphi) \right], \quad (5)$$

where $\kappa^2 = 8\pi/M_p^2$. Equations (4) and (5) imply $\dot{H} = -\frac{1}{2}\kappa^2\dot{\varphi}^2$. The energy density and effective pressure of the inflaton are

$$\rho = \kappa^2 \left(\frac{1}{2}\dot{\varphi}^2 + V \right), \quad p = \kappa^2 \left(\frac{1}{2}\dot{\varphi}^2 - V \right). \quad (6)$$

During slow-roll inflation, the coupled equations (4) and (5) have approximate analytic solutions for the simple potentials. When the value of φ drops low enough, slow-roll ends and the oscillatory regime begins. The approximate analytic forms are (see, e.g., Kaiser 1997, Kofman et al. 1997)

$$\varphi \approx \varphi_{in} \frac{\sin \tau}{\tau} \quad \text{where } \tau = m(t - t_{in}) \text{ and } V = \frac{1}{2}m^2\varphi^2, \quad (7)$$

$$\varphi \approx \frac{\varphi_{in}}{a} \text{cn} \left(\tau, \frac{1}{\sqrt{2}} \right) \quad \text{where } \tau = \sqrt{\lambda}\varphi_{in}(\eta - \eta_{in}) \text{ and } V = \frac{1}{4}\lambda\varphi^4, \quad (8)$$

where $a_{in} = 1$ and cn is a Jacobian elliptic function. The initial values of φ_{in} are $\sim 0.3M_p$ in the quadratic case, and $\sim 0.6M_p$ in the quartic case.

Time-averaging over oscillations shows that $\bar{a} \propto t^{2/3}$ for the quadratic potential, and $\bar{a} \propto t^{1/2}$ for the quartic potential. These are the asymptotic values of the scale factor, but in practice backreaction effects due to couplings will end the preheating oscillations. If one uses the average forms \bar{a} for a , i.e., if one ignores oscillatory behaviour in the inflaton, then one regains the standard results for gravitational wave evolution in the dust and radiation eras. In particular, models with a smooth transition from inflation to radiation-domination, neglecting reheating dynamics, show that there is no super-Hubble amplification of gravitational waves (e.g. Caldwell 1996, Tilley and Maartens 1998). Our numerical integrations show that this averaging loses interesting features in the gravitational waves (see Section 3). We do not use the approximate forms in Eqs. (7) and (8) for our numerical results—instead, we integrate the Friedmann and Klein-Gordon equations numerically.

3. MAXWELL-WEYL GRAVITATIONAL WAVE EQUATIONS

Gravitational wave perturbations in the covariant approach are governed by the Maxwell-Weyl equations:

$$\begin{aligned}(\operatorname{div} E)_\mu &= 0 = (\operatorname{div} H)_\mu, \\ \dot{E}_{\mu\nu} + 3HE_{\mu\nu} - \operatorname{curl} H_{\mu\nu} &= -\frac{1}{2}(\rho + p)\sigma_{\mu\nu}, \\ \dot{H}_{\mu\nu} + 3HH_{\mu\nu} + \operatorname{curl} E_{\mu\nu} &= 0,\end{aligned}$$

where $\sigma_{\mu\nu}$ is the shear, a dot denotes $u^\mu \nabla_\mu$, and div and curl are the covariant spatial divergence and curl for tensors (Maartens and Bassett 1998). These equations hold for perfect fluids and minimally coupled scalar fields (in which case $\rho + p = \kappa^2 \dot{\varphi}^2$). The shear is a tensor potential for the electric and magnetic Weyl tensors (Maartens and Bassett 1998):

$$E_{\mu\nu} = -\dot{\sigma}_{\mu\nu} - 2H\sigma_{\mu\nu}, \quad (9)$$

$$H_{\mu\nu} = \operatorname{curl} \sigma_{\mu\nu}, \quad (10)$$

in close analogy with the Maxwell relations $\vec{E} = -\vec{\nabla} \vec{A}$ and $\vec{H} = \operatorname{curl} \vec{A}$. Taking the curl and dot of the Maxwell-Weyl propagation equations, and using the identity for the tensor curl of the curl, we find wave equations for the three tensors (Dunsby et al. 1997, Maartens and Bassett 1998, Challinor 2000). Using equations (4) and (5), these become

$$\ddot{E}_{\mu\nu} + 7H\dot{E}_{\mu\nu} + 4\kappa^2 V(\varphi)E_{\mu\nu} = \Delta E_{\mu\nu} + \kappa^2 \dot{\varphi} [5H\dot{\varphi} + V'(\varphi)] \sigma_{\mu\nu}, \quad (11)$$

$$\ddot{H}_{\mu\nu} + 7H\dot{H}_{\mu\nu} + 4\kappa^2 V(\varphi)H_{\mu\nu} = \Delta H_{\mu\nu}, \quad (12)$$

where Δ is the covariant spatial Laplacian. One can also derive a wave equation for the shear:

$$\ddot{\sigma}_{\mu\nu} + 5H\dot{\sigma}_{\mu\nu} + \kappa^2 [2V(\varphi) - \frac{1}{2}\dot{\varphi}^2] \sigma_{\mu\nu} = \Delta \sigma_{\mu\nu}. \quad (13)$$

We decompose the tensors into modes (Challinor 2000)

$$E_{\mu\nu} = a^{-2} \sum k^2 [\mathcal{E}_k Q_{\mu\nu}^{(k)} + \tilde{\mathcal{E}}_k \tilde{Q}_{\mu\nu}^{(k)}], \quad (14)$$

$$H_{\mu\nu} = a^{-2} \sum k^2 [\mathcal{H}_k Q_{\mu\nu}^{(k)} + \tilde{\mathcal{H}}_k \tilde{Q}_{\mu\nu}^{(k)}], \quad (15)$$

$$\sigma_{\mu\nu} = a^{-1} \sum k [\mathcal{S}_k Q_{\mu\nu}^{(k)} + \tilde{\mathcal{S}}_k \tilde{Q}_{\mu\nu}^{(k)}], \quad (16)$$

where \sum denotes a symbolic sum over harmonic modes, and Q , \tilde{Q} are tensor harmonics of electric and magnetic parity, which are time-independent, transverse traceless, and related by

$$\operatorname{curl} Q_{\mu\nu}^{(k)} = \frac{k}{a} \tilde{Q}_{\mu\nu}^{(k)},$$

showing that the different polarization states are coupled. The mode functions \mathcal{E}_k determine the tensor contribution to the CMB power spectrum. The magnetic Weyl mode functions are algebraically related to the shear mode functions by

$$\mathcal{H}_k = \tilde{\mathcal{S}}_k. \quad (17)$$

Specializing the results in Challinor (2000) to the scalar field case, we find the evolution equations

$$\dot{\mathcal{E}}_k + 3H\mathcal{E}_k - \frac{a}{k} \left(\frac{k^2}{a^2} - \frac{1}{2}\kappa^2 \dot{\varphi}^2 \right) \mathcal{S}_k = 0, \quad (18)$$

$$\dot{\mathcal{S}}_k + H\mathcal{S}_k + \frac{k}{a} \mathcal{E}_k = 0. \quad (19)$$

For comparison, in the time-averaged approximation, the $\dot{\varphi}^2$ term in the coefficient of \mathcal{S}_k in Eq. (18) is replaced by $\gamma\rho$, where $\rho = \rho_{\text{in}} a^{-3\gamma}$, with $\gamma = 1$ for the averaged quadratic case (i.e. cold matter or ‘dust’), and $\gamma = \frac{4}{3}$ for the averaged quartic case (i.e. radiation).

4. NUMERICAL RESULTS

Equations (17)–(19) for the gravitational wave perturbations, and (4)–(5) for the background are integrated numerically, and the results are compared with those for the time-averaged approximation. We investigated the evolution of COBE modes, which left the Hubble radius at about 50 to 60 e-folds before the end of inflation, so that $k/aH \sim 10^{-24}$ at the start of preheating, $t = t_{in}$. We also integrated for a typical small scale mode, with $k/aH \sim 10$ at $t = t_{in}$. The initial values for φ were given in the previous section. We used the slow-roll relation to set the initial inflaton velocity, $\dot{\varphi}_{in} = -M_p V_\varphi / \sqrt{24\pi V}$. For the tensor modes, we used the initial value 10^{-5} . The inflaton mass and self-coupling were chosen as $m = 10^{-6} M_p$ and $\lambda = 10^{-12}$. In order to take account of backreaction effects, which will end the coherent inflaton oscillations, we terminated the integrations after $\tau = 100$.

The results are shown in Figs. 1–8.

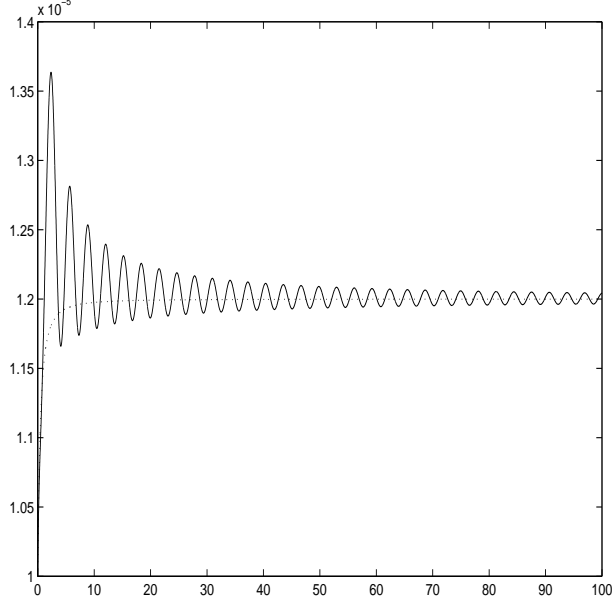


FIG. 1. The electric Weyl mode $\mathcal{E}_k(\tau)$ on super-Hubble scales, $k/(aH)_{in} = 10^{-24}$. The solid line is for the inflaton potential $V = \frac{1}{2}m^2\varphi^2$, while the dotted line is for the corresponding time-averaged model, which behaves like cold matter (dust).

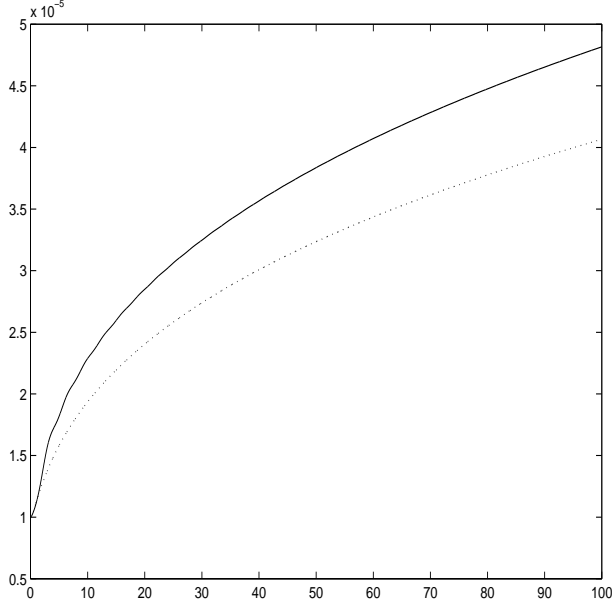


FIG. 2. The magnetic Weyl mode $\mathcal{H}_k(\tau)$, as for Fig. 1.

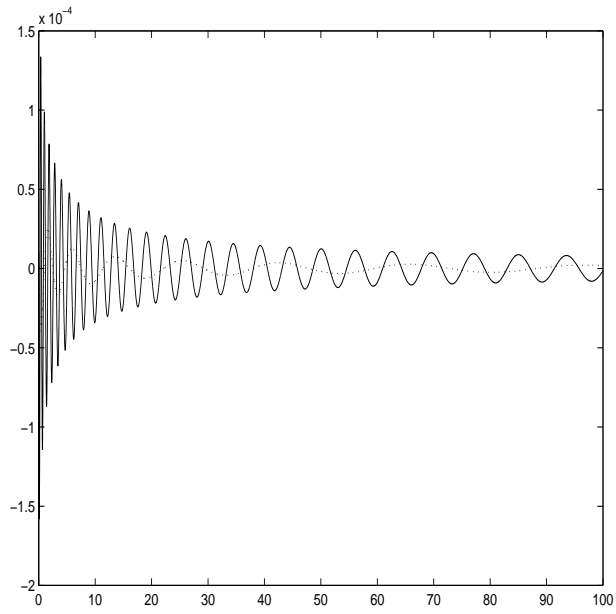


FIG. 3. The electric Weyl mode $\mathcal{E}_k(\tau)$ on sub-Hubble scales, $k/(aH)_{in} = 10$. The solid line is for the inflaton potential $V = \frac{1}{2}m^2\varphi^2$, while the dotted line is for the corresponding time-averaged model, which behaves like cold matter (dust).

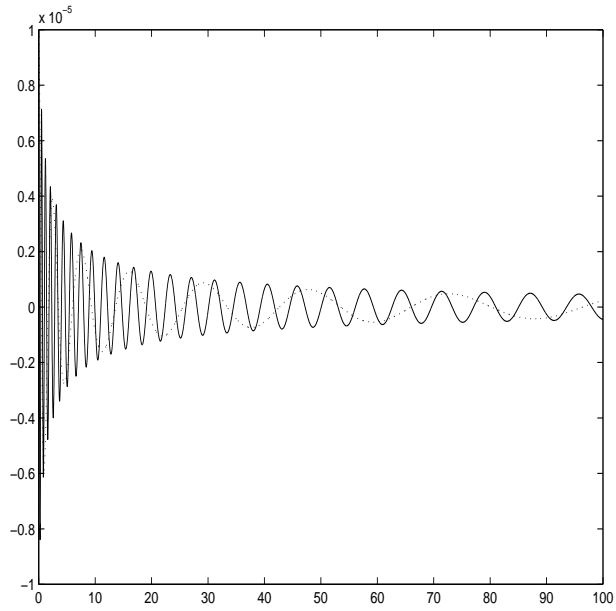


FIG. 4. The magnetic Weyl mode $\mathcal{H}_k(\tau)$, as for Fig. 3.

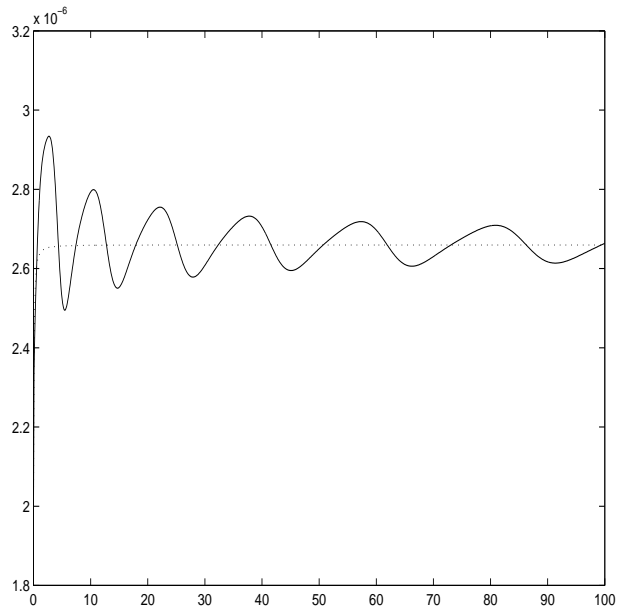


FIG. 5. The electric Weyl mode $\mathcal{E}_k(\tau)$ on super-Hubble scales, $k/(aH)_{in} = 10^{-24}$. The solid line is for the inflaton potential $V = \frac{1}{4}\lambda^4\varphi^4$, while the dotted line is for the corresponding time-averaged model, which behaves like radiation.

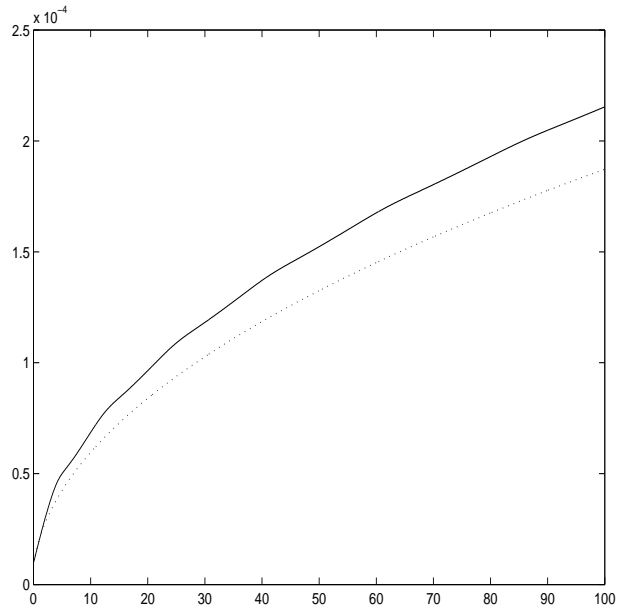


FIG. 6. The magnetic Weyl mode $\mathcal{H}_k(\tau)$, as for Fig. 5.

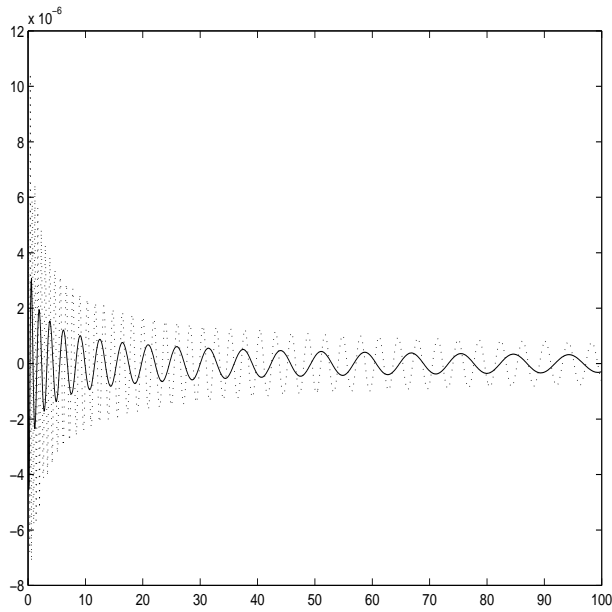


FIG. 7. The electric Weyl mode $\mathcal{E}_k(\tau)$ on sub-Hubble scales, $k/(aH)_{in} = 5$. The solid line is for the inflaton potential $V = \frac{1}{4}\lambda^4\varphi^4$, while the dotted line is for the corresponding time-averaged model, which behaves like radiation.

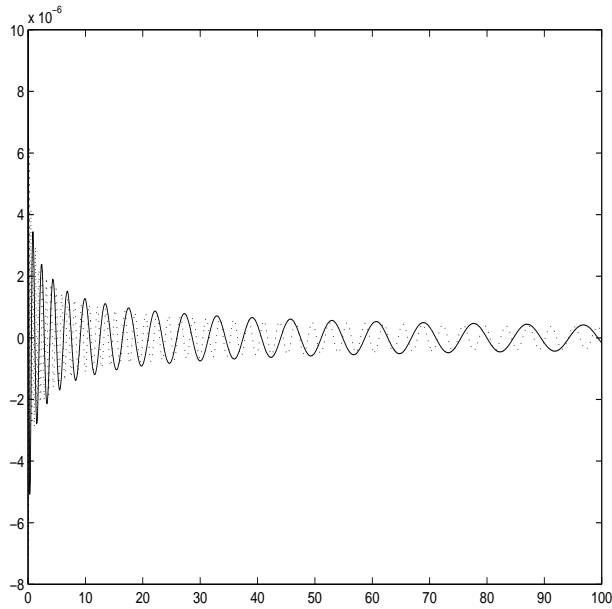


FIG. 8. The magnetic Weyl mode $\mathcal{H}_k(\tau)$, as for Fig. 7.

5. CONCLUSIONS

Apart from specific features peculiar to the potential, these figures reflect two key properties: (1) Super-Hubble gravitational waves are not significantly amplified by coherent inflaton oscillations during preheating; (2) gravitational waves on all scales carry some imprint of the coherent oscillatory dynamics of the inflaton during preheating. The latter point is clearly brought out by a comparison with the evolution in the absence of oscillations, i.e. for the time-averaged scale factor.

In particular, one can see that the electric Weyl modes on super-Hubble scales, which determine the effect of gravitational waves on the CMB (Challinor 2000), inherit oscillations from the inflaton. In the early part of preheating, there is also some amplification on average of \mathcal{E}_k , so that if backreaction takes effect early, then this does produce a

small amplification relative to the no-oscillation time-averaged model. However, one cannot expect these preheating imprints on \mathcal{E}_k to lead to detectable differences in the CMB power spectrum. Firstly, the oscillations will effectively be averaged out, and secondly, any amplification is likely to be scale invariant for all measurable anisotropies, since the Hubble scale at preheating corresponds to about 1 metre today, so that all cosmologically significant scales at preheating effectively have $k = 0$, and behave like the particular scale chosen in our numerical integrations (Bassett et al. 2000, Jedamzik and Sigl 2000).

Gravitational waves on sub-Hubble scales oscillate even in the case of time-averaged background, but Figs. 3, 4, 7 and 8 show that the frequency and amplitude of oscillation are significantly modulated by the inflaton oscillations. Thus preheating leaves an imprint on these scales. In principle, this could be detected, but in practice the signal will be far too weak, since there is no preheating amplification on these scales.

The absence of amplification is fundamentally due to the expansion of the universe, since there is strong amplification on a Minkowski background of gravitational waves during preheating (Bassett 1997, Tilley 2000).

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